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## On the rapid computation of various polylogarithmic constants

David Bailey, Peter Borwein and Simon Plouffe

**Abstract.** We give algorithms for the computation of the  $d$ -th digit of certain transcendental numbers in various bases. These algorithms can be easily implemented (multiple precision arithmetic is not needed), require virtually no memory, and feature run times that scale nearly linearly with the order of the digit desired. They make it feasible to compute, for example, the billionth binary digit of  $\log(2)$  or  $\pi$  on a modest work station in a few hours run time. We demonstrate this technique by computing the ten billionth hexadecimal digit of  $\pi$ , the billionth hexadecimal digits of  $\pi^2$ ,  $\log(2)$  and  $\log^2(2)$ , and the ten billionth decimal digit of  $\log(9/10)$ . These calculations rest on the observation that very special types of identities exist for certain numbers like  $\pi$ ,  $\pi^2$ ,  $\log(2)$  and  $\log^2(2)$ . These are essentially polylogarithmic ladders in an integer base. A

number of these identities that we derive in this work appear to be new, for example the critical identity for  $\pi$ :

$$\pi = \sum_{i=0}^{\infty} \frac{1}{16^i} \left( \frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6} \right).$$

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