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Computational Classification of Numbers and Algebraic Properties

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1 Introduction

In this paper, we propose a computational classification of finite characteristic numbers (Laurent series with coefficients in a finite field), and prove that some classes have good algebraic properties. This provides tools from the theories of computation, formal languages, and formal logic for finer study of transcendence and algebraic independence questions. Using them, we place some well-known transcendental numbers occurring in number theory in the computational hierarchy.

Existence of or lack of patterns in the digit sequences of naturally occurring real numbers is a natural question. Rational numbers (and only rational numbers) have eventually periodic digit sequences. But the question has not been studied much for irrational real numbers, except for statistical studies on normality and randomness of digits for general numbers as well as special numbers such as π. Apart from the fact that no interesting patterns are found in general, the other reason for the lack of such studies is that irrationals usually are not naturally presented by their digit expansions (say decimals), at least in theoretical studies. Also, since it is hard to control carry-overs well, when we add or multiply, it is usually hard to manipulate the formulas to get good control on digit expansions of sums and products.

The situation seems to be much better for finite characteristic numbers: There are well-known strong analogies between integers, rationals, and reals on one hand, and polynomials, rational functions, and Laurent series (all with coefficients in a finite field) on the other. Again rational functions correspond to eventually periodic Laurent series expansions. Also the Laurent series representation is widely used: There are no

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